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APPLICATION OF THE ADAPTIVE VIEWPOINT TO A NONLINEAR SYSTEM WITH TIME VARYING PARAMETERS

by

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Dean of Graduate College

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Ames, Iowa

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INTRODUCTION

We live in a nonlinear world, but very frequently we base our understanding of it on linear mathematical models which fail to predict (or even hint at) some of the most interesting physical phenomena which are readily discernible in nature every day. Jump phenomena, sub-harmonic oscillations, limit cycles, and frequency entrainments are just a few examples of physical phenomena which are inadequately described by linear mathematical models.

A much more serious consequence than our failure to predict existing nonlinear physical phenomena with our linear mathematical models is our resultant tendency to think and synthesize in terms of linear components and devices when designing systems to perform a given task. Quite often a nonlinear system might be more reliable, more efficient, more economical, simpler, and, in general, more suitable than the corresponding linear system. Unfortunately, our "linear training" teaches us to be linear.

To state that it is easier to analyze a linear mathematical model described by linear integral-differential equations with constant coefficients is not a valid reason for specifying linear systems. Nor is our ignorance in the field of nonlinear mathematics a valid excuse. Nor is it valid to say that many physical systems may be approximated by a linear mathematical model because nearly all observed deviations from predicted results may be attributed to our failure to take into account the physical nonlinearities.

The purpose of this dissertation is to use the adaptive viewpoint and to present the steps leading to the synthesis of a physical system with a specified transfer characteristic and subject to a given set of specifications. It will be shown that the specified transfer characteristic may be interpreted to be a so-called open-loop system, a closed-loop system, or an adaptive control system depending on the viewpoint that is taken. Although in some cases the same equation (or set of equations) may describe the terminal characteristics, the actual internal physical configuration may be radically different. The optimum physical configuration is the one then that maximizes the effect of the desirable characteristics of the components selected and minimizes the effects of their undesirable characteristics. For example, simple feedback makes it possible to "barter" a higher than necessary (but somewhat varying gain) for a lower but more constant one. Similarly, the use of the adaptive viewpoint will make it possible to reduce certain stability problems if the specified performance is not demanded immediately. Or to state this another way, system performance may actually improve with age.

Furthermore, this dissertation will show that the choice of an adaptive control system will make it possible to correct for undesirable changes in components in a manner which is superior to the conventional feedback system. It is superior because it may not cause the stability problems which arise quite frequently in conventional feedback systems with high loop gains. Unfortunately, when stability problems do arise in an adaptive control system, they are of a much more complex nature and, at the present time, are not as well understood as those in linear control systems.

A logical starting point for this discussion is a review of existing literature and, in particular, a statement and an agreement on an acceptable definition of what constitutes an adaptive control system.

REVIEW OF LITERATURE

At the present time there is no universally accepted definition for an adaptive control system. Therefore, it is not surprising that there is also no universally accepted classification for the various types of existing adaptive control systems. Consequently, our first task is to attempt to state a definition which is compatible with the majority of existing definitions. Then, hopefully, with this judiciously chosen definition, all existing systems may be conveniently classified using this definition.

Definitions of an Adaptive Control System

The most concise definition of an adaptive control system is given by Truxal (8) who states that "An adaptive control system is one which is designed with an adaptive viewpoint". Truxal elaborates, however, "By this adaptive viewpoint one obtains a logical, simple, and straightforward technique toward the inclusion of a nonlinear element within the system to obtain some reasonable performance specifications or meet some reasonable optimization criteria". Whitaker (9, p. 3) states that "An adaptive system is one that adapts itself to a changing environment, a changing character of input signals, or a changing system or component characteristic in such a manner that a desired performance will be maintained". Anderson *et al.* (1) feel that "the concept of the self-adaptive control system is based on the premise that either implicitly or explicitly such a system must perform the operations of a) continuous measurement of system dynamic performance; b) continuous evaluation of performance on the basis of some predetermined criterion; and c) continuous readjustment of system parameters for optimum system performance in accordance with the measurement and the evaluation performed". Galbiati (3, p. 6) states that "An adaptive control system is a system having some essential parameter affected by a variation in at least one environmental factor input signal and also containing a means of compensating for the variation of the parameter".

Even though many more definitions could be included here, it appears sufficient to include just one more by quoting Aseltine (2) who states "I think you need three things in this design of an adaptive system. First you must have a measure of system performance while the system is operating; second, you must have a means of converting this measure of performance into numbers or some measure of how good the performance is; and then finally, you must have a means of using this number to change the system itself".

For purposes of this dissertation a system is <u>defined</u> to be an adaptive control system if it meets <u>all</u> <u>three</u> of the following conditions:

1. <u>System performance must be determined</u>. This determination may be made by observing system response to actual command inputs, noise inputs (both natural and manmade) or special inputs such as impulses and sine waves. A judiciously chosen limit cycle may also be used.

2. <u>Observed performance must be evaluated</u>. This evaluation is most frequently done by comparing it to the desired performance. The desired performance is an embodiment of the system specifications in one form or another such as, for

example, a model of the desired system.

3. The results of the evaluation must be used to modify some part of the system. The parts most frequently modified are system gains, time constants, values of resistors and capacitors, and, in general, anything that can undergo a controllable change.

Classification of Adaptive Control Systems

It would seem natural and convenient to classify all existing adaptive control systems according to the requirements stated in the definition given in the preceding section. Unfortunately, this is not possible in all cases because there is a class of systems which does not satisfy all three of the conditions in the stated definition, but, nevertheless, is classed as adaptive by some authorities, but not all. This class will be designated in this section as quasi-adaptive. By introducing this fourth classification of quasi-adaptive, it is possible to discuss all types of existing adaptive control systems under the following four categories:

1. Measurement of system performance
2. Evaluation of measured performance
3. Change in system parameters
4. Quasi-adaptive systems.

Measurement of system performance

System performance and/or system transfer characteristic may be measured by means of a test signal, a limit cycle, or cross-correlation between the output and the input. Test signals may be sinusoidal, a series of generated impulses, or white noise (both natural occurring and manmade).

An impulse-excited adaptive system was studied on an analog computer by Aseltine *et al.* (2). Their system was essentially a second-order system with the damping ratio ζ adjusted by the adaptive loop which utilizes an area-ratio figure of merit applied to the output resulting from a series of unit impulse inputs generated by an external pulse train generator. A typical example of a random (white noise) test signal has been studied by Anderson *et al.* (1). Roberts (6) used the amplitude and frequency of a natural occurring limit cycle to determine the characteristics of his system. Anderson *et al.* (1) cross-correlated the output and the input of a system excited by a noise signal to obtain one point on the impulsive response of the system. With twelve channels of digital cross-correlation, each having a different delay, twelve points on the impulsive response were obtained. The underlying assumption was simply that the noise of the signal input has a bandwidth much larger (at least 10 times) than that of the physical system. This assumption is easily satisfied in most conventional control systems.

Evaluation of system performance

Ostensibly a system is built to serve a purpose; the engineering statement of this purpose is called a specification; and the comparison of the actual observed performance to the specified performance involves an evaluation and an error criterion. There are numerous types of error criteria currently used in the evaluation of the performance of adaptive control systems. Sarture and Aseltine (7) define and explain all of the commonly used ones. It may be informative to list a few as follows: impulse response area ratio (IRAR), integrated absolute value of the error

(IAE), integrated squared value of the error (ISE), integrated time multiplied by absolute value of error (ITAE), mean square (MS) error, and root mean square (RMS) error.

Change in system parameters

The two system parameters which are usually changed to make a system adaptive are gain and the position of select poles and zeros. Gain changes may be made either continuously or discretely, for example, 10 steps from maximum to minimum gain. The change in the positions of certain poles and zeros may also be continuous or discrete. All of the above cases are discussed in considerable detail by Galbiati (3).

Quasi-adaptive systems

Quasi-adaptive systems are those which are classed as adaptive by some authorities and non-adaptive by others because of conflicting definitions. In general, quasi-adaptive systems reduce the effect of unavoidable variations in system parameters by inherent design using fixed components in ingenious configurations, such as feedback of signals, rather than controlled deliberate parameter changes. Most present day quasi-adaptive systems may be divided into two classes: programmed quasi-adaptive and input quasi-adaptive.

A programmed quasi-adaptive system is one in which situations which cause a deterioration in performance are known beforehand and means are taken to change system parameters so as to reduce this deterioration. A typical system with programmed temperature correction is the case of the transistor amplifier with a thermistor for thermal stabilization. The "programming" in this example is the prior selection and installation of a thermistor with the appropriate compensating temperature characteristic to offset the known transistor temperature sensitivity. This system is not truly adaptive because actual performance is not measured.

In an input quasi-adaptive system, some characteristic of the input signal is used to change a system parameter. A typical example may be found in the system proposed by Keiser (5) where the adjustment of the system parameters is made on the basis of measurements of the short time auto-correlation of the signal plus noise at the input. It does not have the advantages of a truly adaptive system because the changes are made essentially open-loop and are not dependent on the actual performance measured at the output.

METHOD OF ATTACK

As the title of this dissertation indicates, the area of investigation is primarily that one which concerns itself with the application of the adaptive viewpoint to a class of physical systems which can be described by a nonlinear equation (or equations) with time varying parameters. Or to state this another way, the basic problem is to determine all possible characteristics and consequences of using the adaptive viewpoint to synthesize a system which satisfies certain given specifications.

The first and most important step in solving any engineering problem is to define the problem. Trivial as this may seem, the author is personally aware of several engineering projects where this was not done.

Statement of Problem

Let it be assumed that it is necessary to synthesize a system which is to have a transfer characteristic of 10. The term "transfer characteristic" is defined to be the ratio of the output to the input of the system. It might have the units of radians per volt, foot-pounds per volt, radians per second per ampere, or even volts per volt and be dimensionless, as in the case of a voltage amplifier. Simulation on an analog computer would also result in a dimensionless ratio.

If the numerical value of the transfer characteristic is to be <u>exactly</u> 10, constant for all time, all possible magnitudes, and variations of the input signal, then the solution to this problem is not physically realizable because no components or configuration of existing components has been discovered up to the present time which has these characteristics. A more realistic, and far more typical set of specifications, would be 10 ± 1 per cent for an input variation over the range of 10 to 1.

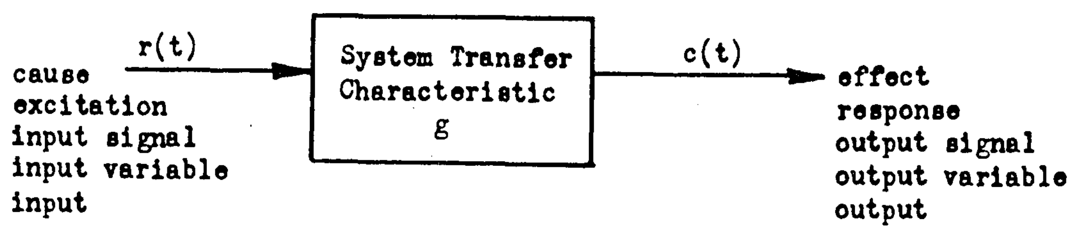
Synthetical Approaches

There are three types of problems in system engineering, i.e., the analysis problem, the synthesis problem, and the instrumentation problem. Although they appear to be very closely related, the amount of effort and ingenuity required to solve them is radically different. The similarities and differences among the three types of problems can best be explained by the diagram shown in Figure 1 where the input variable is r(t), the output variable is c(t), and the system transfer characteristic is represented by the lower case "g". The term "transfer function" has not been used and has been intentionally avoided because common and repeated usage has given it the definition of being the ratio of the Laplace transform of the output variable divided by the Laplace transform of the input variable.

Referring again to Figure 1, when r(t) and g are known, and c(t) is to be determined, this is called the analysis problem. When c(t) and g are known, and r(t) is to be determined, this is the instrumentation problem. Finally, in this dissertation, it is assumed that the character of both r(t) and c(t) is known, and it is necessary to determine the system transfer characteristic g. In general, there is not a unique solution to the synthesis problem so it is not surprising that it is the least understood of the three problems.

The use of the adjective "synthetical" may seem to be a little strange, but it is exactly analogous to the use of the adjective "analyti-

Figure 1. Block diagram illustrating the vocabulary commonly associated with the description of a system or situation


r(t)
cause
excitation
input signal
input variable
input
System Transfer
Characteristic
g
c(t)
effect
response
output signal
output variable
output

cal" to describe approaches to the solving of the analysis problem, as, for example, the "analytical approach".

The three approaches commonly used today to solve the system synthesis problem may best be described by the following terms: the open-loop system approach, the closed-loop system approach, and the adaptive control system approach. The word "viewpoint" is sometimes used instead of approach as, for example, the adaptive control system viewpoint.

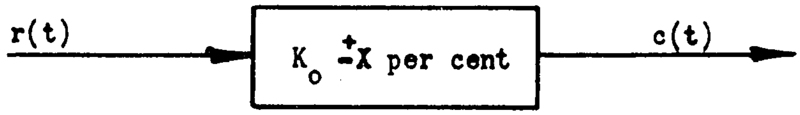
Open-loop system approach

Suppose that for simplicity it is assumed that the desired transfer characteristic is to be 10 ± 1 per cent and dimensionless. More specifically, it could be a voltage amplifier with units of volts per volt, but this does not affect this discussion since the philosophy of the various approaches is the same as if it included an electromechanical actuator, such as a motor, and had the units of torque per volt, displacement per unit current, or velocity per unit angle.

To design a voltage amplifier of 10 is not a difficult problem. Conventional vacuum tubes and transistors may be used in a common cathode or common emitter configuration respectively, to realize this value. Why then is this a problem? It is a problem because no components are ever supplied which have exactly the value stated. Typical tolerances are ± 5 per cent for resistors, ± 10 per cent for capacitors, and ± 20 per cent and even higher for vacuum tubes and transistors. The gain of this amplifier could very easily be 10, but the ± 1 per cent specification would not be satisfied.

A typical open-loop system is shown in Figure 2 which shows that the

Figure 2. Block diagram of a typical open-loop system


r(t)
$K_o \pm X$ per cent
c(t)

tolerance on the ratio of the output to the input is equal in magnitude to the tolerance in the block. Since the desired tolerance is ± 1 per cent and the block tolerance in this typical case might be about ± 10 per cent, this open-loop system does not meet the given specifications. Figure 3 shows a typical set of transfer characteristics for an open-loop system when K_o varies over a range by a factor of 10 to 1 higher and lower than its nominal value of 10.

Closed-loop system approach

A closed-loop feedback control system is defined by Grabbe et al. (4, p. 19-06) to be a "control system which tends to maintain a prescribed relationship of one system variable to another by comparing functions of these variables and using the relationship as a means of control". Figure 4 shows a block diagram of a typical feedback control system with standard symbols and their values for the problem under discussion.

Suppose now that h is equal to 0.09 and g is equal to K_2 which undergoes the same variation that K_o did in the previous section. Plots of c(t) versus r(t) are shown in Figure 5 and have been calculated using the well known feedback formula $c/r = g/(1 + gh)$. Comparison of the curves shown in Figure 5 with those shown in Figure 4 shows that the same per cent variation in K_o and K_2 results in much less variation in the transfer characteristic $c(t)/r(t)$ in the latter case than in the former.

Adaptive control system approach

One of the many block diagram configurations which satisfies the definition of an adaptive control system given in this dissertation is shown in

Figure 3. Transfer characteristic for a typical open-loop system such as shown in Figure 1

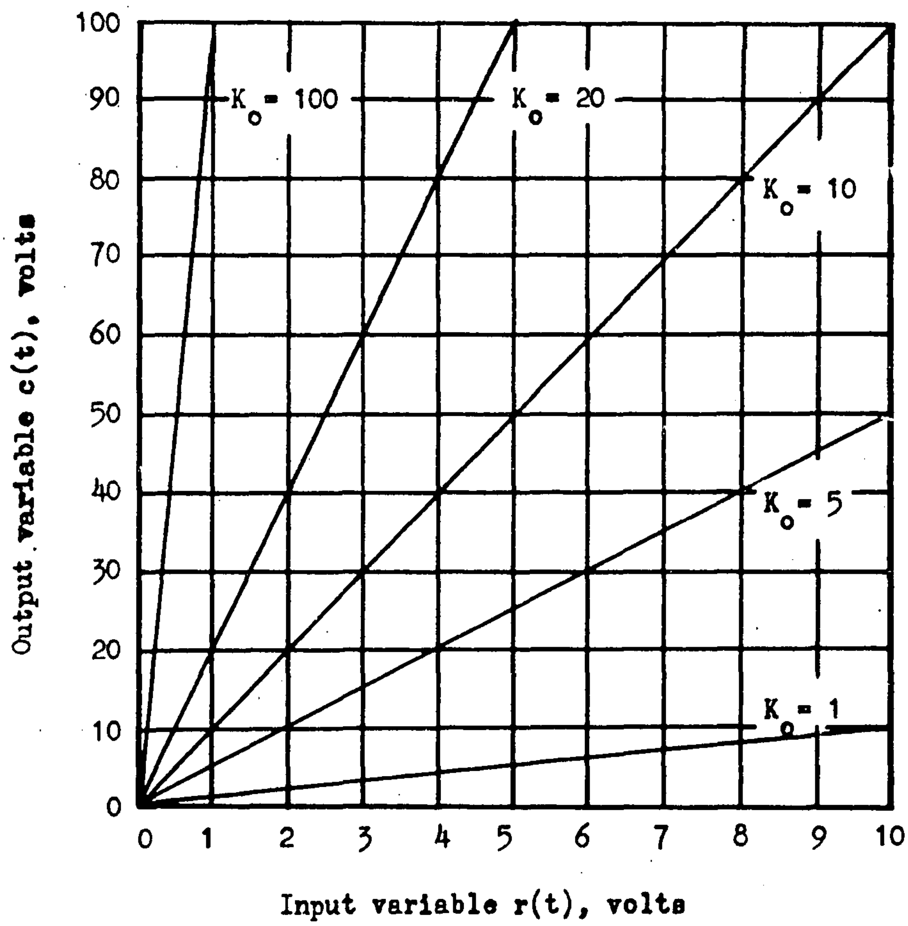

100
90
80
70
60
50
40
30
20
10
0
0
1
2
3
4
5
6
7
8
9
10
K_o = 100
K_o = 20
K_o = 10
K_o = 5
K_o = 1
Output variable c(t), volts
Input variable r(t), volts

Figure 4. Block diagram of a typical feedback control system with standard symbols and their values for the problem under discussion

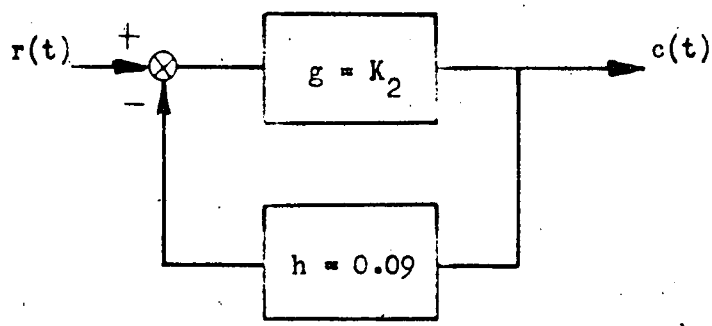

r(t)
+
−
g = K2
c(t)
h = 0.09

Figure 5. Transfer characteristic for a typical closed-loop feedback control system such as shown in Figure 4

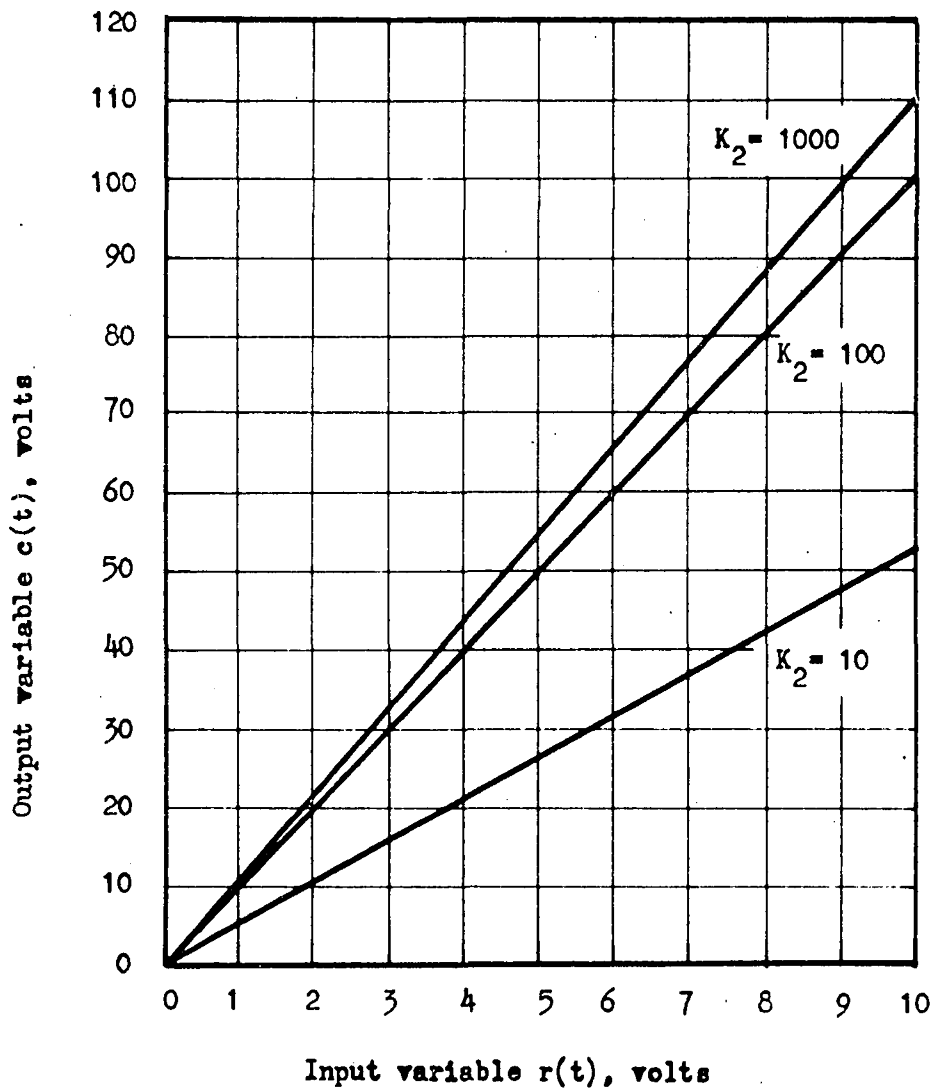

120
110
100
90
80
70
60
50
40
30
20
10
0
Output variable c(t), volts
K_2= 1000
K_2= 100
K_2= 10
0
1
2
3
4
5
6
7
8
9
10
Input variable r(t), volts

Figure 6. Block diagram of a typical reciprocal model reference adaptive control system represented by the equation $c = K_{a1} + K_1(r - c/K_d)\ r$

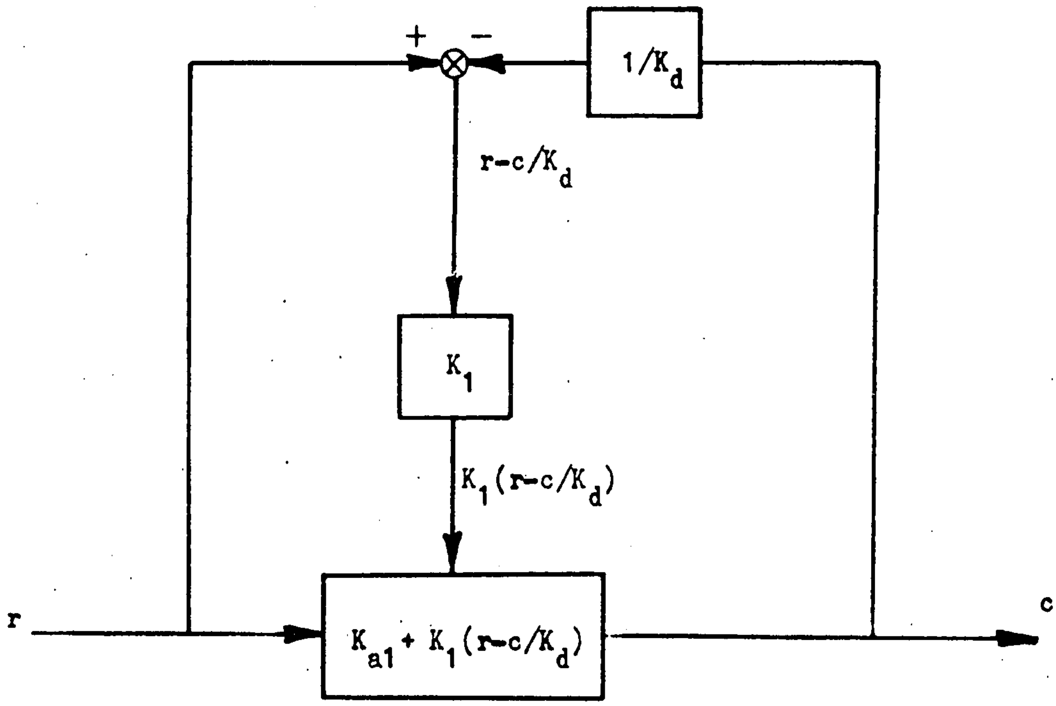

+
−
1/K_d
r−c/K_d
K_1
K_1(r−c/K_d)
r
K_{a1} + K_1(r−c/K_d)
c

Figure 6. More specifically, it may be classed as a reciprocal model reference adaptive control system. The measurement of system performance is accomplished by multiplying the output by the reciprocal of the desired transfer characteristic; the evaluation is accomplished by comparing this function of the output to the input; and the change is the increment added to (or subtracted from) the nominal, but variable, gain K_{a1}.

Referring to the block diagram shown in Figure 6, the equation relating the output variable c(t) and the input variable r(t) may be written as[1]

$$c = \left[K_{a1} + K_1(r - c/K_d)\right] r \qquad 1$$

rearranging Equation 1

$$c + \frac{K_1}{K_d} rc = r(K_{a1} + K_1 r) \qquad 2$$

solving for c provided that $(1 + \frac{K_1}{K_d} r) \neq 0$

$$c = \frac{r(K_{a1} + K_1 r)}{(1 + \frac{K_1}{K_d} r)} \qquad 3$$

An alternate form of Equation 1 which is sometimes more convenient may be obtained by dividing both numerator and denominator by K_1 provided that K_1 does not equal zero.

$$c = \frac{r(\frac{K_{a1}}{K_1} + r)}{(\frac{1}{K_1} + \frac{1}{K_d} r)}, \; K_1 \neq 0. \qquad 4$$

---

[1]Henceforth, for convenience, c(t) will be written as simply c, and r(t) as r, respectively.

It is now interesting to consider two cases.

Case 1. Let $K_{a1} = K_d$. This is the special case when the nominal gain K_{a1} is equal to the desired gain K_d. Substituting this value for K_{a1} into either Equation 1 or 3

$$c = K_d r \qquad 5$$

which is the desired relationship.

Case 2. Let $K_1 r \gg K_{a1}$ and $K_1 r \gg K_d$. This is the case when the adaptive loop gain is very high. Then from Equation 4

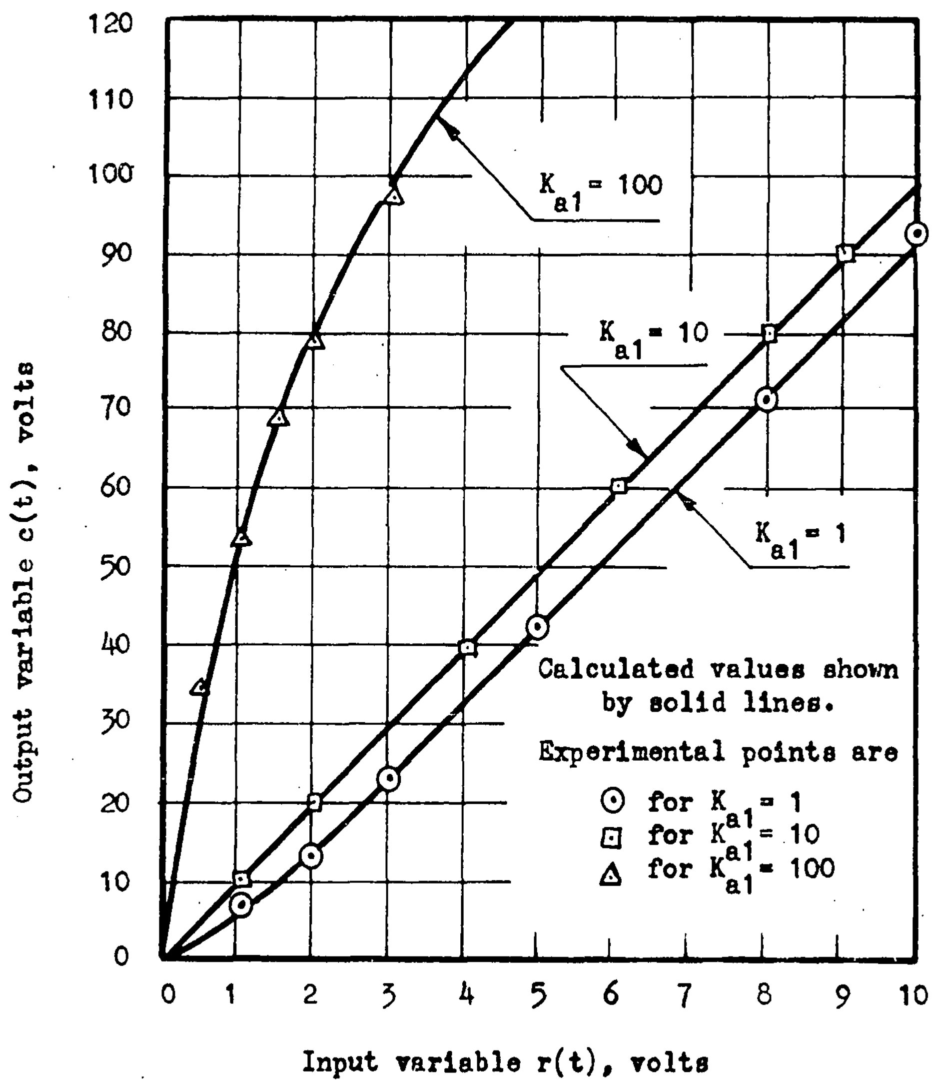
$$c \approx K_d r \qquad 6$$

which is again the desired relationship.

Before becoming too elated with the above results, it should be noted that the fundamental equations are indeed nonlinear and as a consequence have some of the idiosyncrasies associated with nonlinear equations. For example, the condition that $(1 + \frac{K_1}{K_d} r) \neq 0$ is not just a mathematical frill, because in the analog computer simulation of this equation with $K_1 = 10$, $r = -1$, and $K_d = 10$, the output actually is indeterminate and takes on almost any value of voltage with the +100 and -100 volt amplifier saturation voltages being about equally probable.

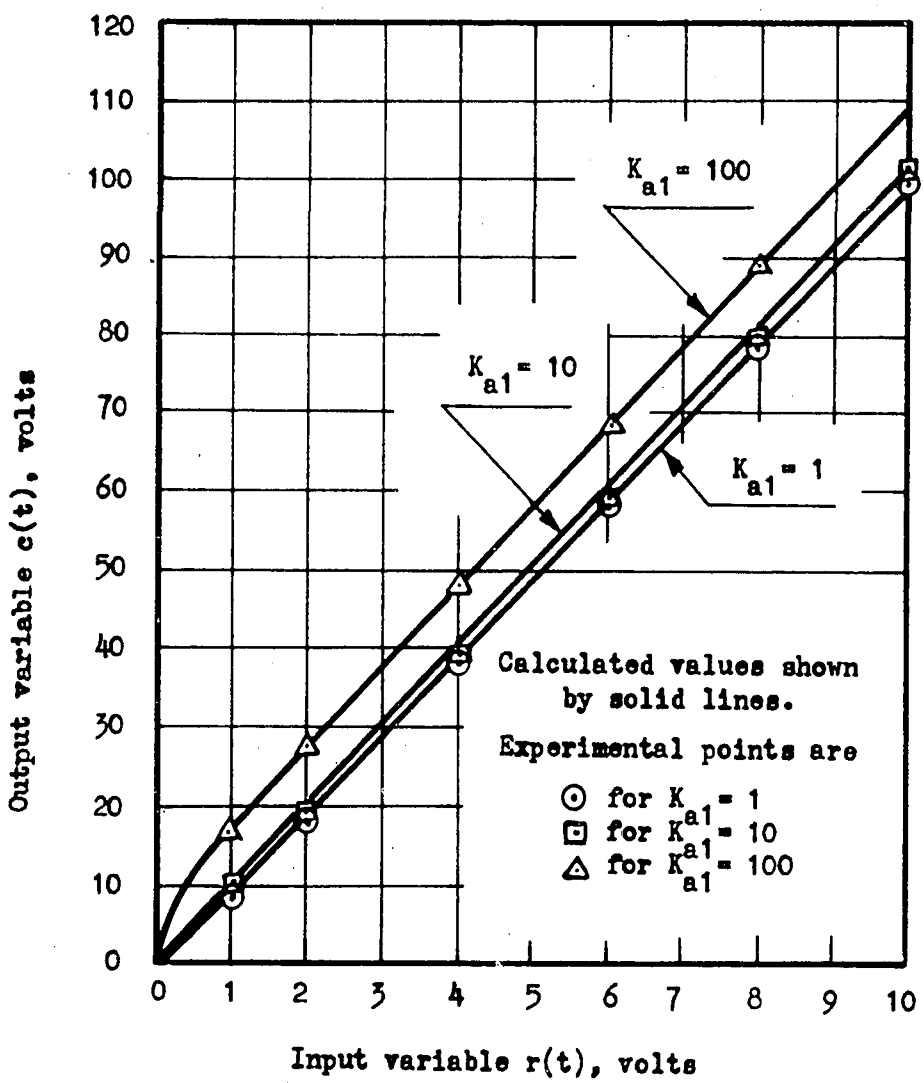
Because Equation 4 is nonlinear, it is most informative to plot c as a function of r for various values of K_{a1} and K_1. To be more specific, Figure 7 is a plot of Equation 4 for $K_d = 10$, $K_1 = 10$, and K_{a1} having the values of 1, 10, and 100. Figure 8 is the same except $K_1 = 100$ instead of 10; and in Figure 9, K_1 has the value of 1000. The solid lines represent the calculated values, while the small circles, squares, and triangles represent experimentally observed data points obtained by simulation on a

Figure 7. Plots of Equation 3 $c = \frac{r(K_{a1} + K_1 r)}{(1 + \frac{K_1 r}{K_d})}$ with $K_d = 10$ and $K_1 = 10$



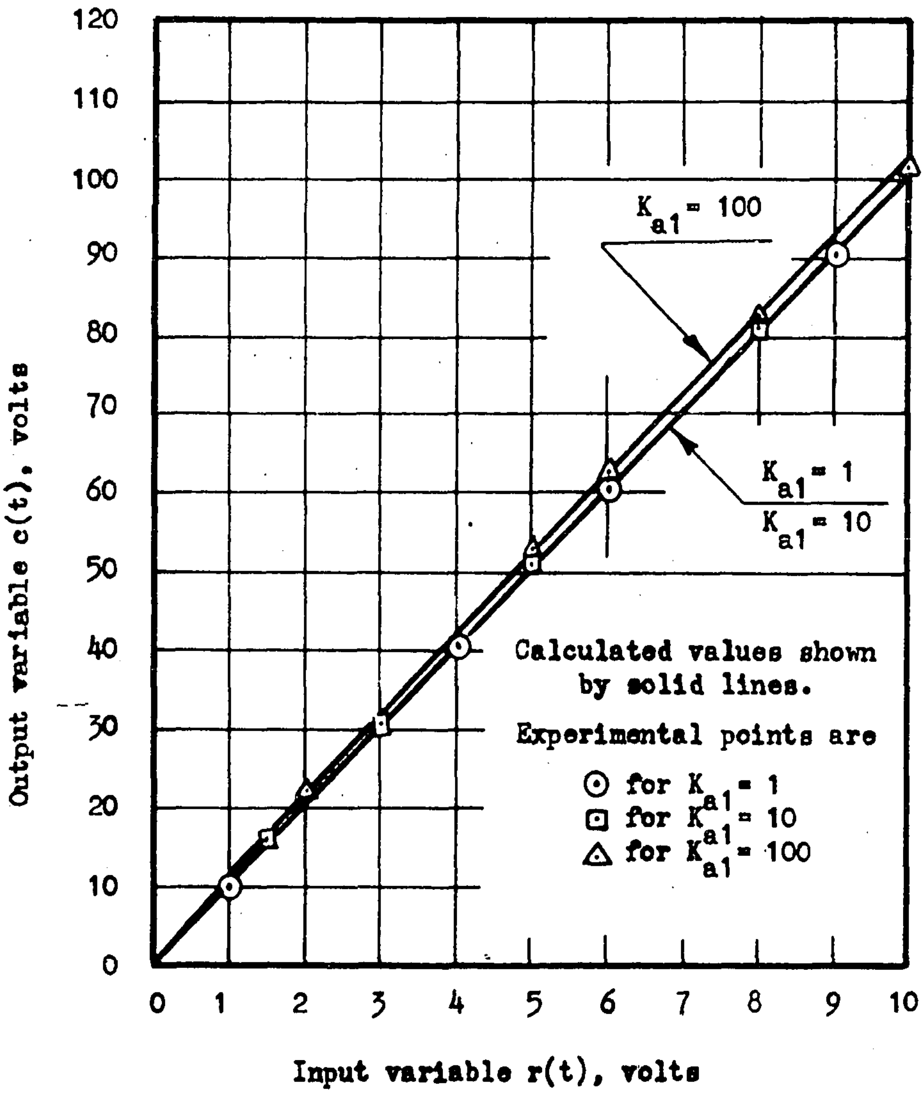
120
110
100
90
80
70
60
50
40
30
20
10
0
0 1 2 3 4 5 6 7 8 9 10
Output variable c(t), volts
Input variable r(t), volts
$K_{a1} = 100$
$K_{a1} = 10$
$K_{a1} = 1$
Calculated values shown by solid lines.
Experimental points are
⊙ for $K_{a1} = 1$
⊡ for $K_{a1} = 10$
△ for $K_{a1} = 100$

Figure 8. Plots of Equation 3 $c = \dfrac{r(K_{a1} + K_1 r)}{(1 + \frac{K_1 r}{K_d})}$ with $K_d = 10$ and $K_1 = 100$



K_{a1} = 100
K_{a1} = 10
K_{a1} = 1
Calculated values shown by solid lines.
Experimental points are
⊙ for K_{a1} = 1
⊡ for K_{a1} = 10
△ for K_{a1} = 100
Output variable c(t), volts
Input variable r(t), volts
0
10
20
30
40
50
60
70
80
90
100
110
120
1
2
3
4
5
6
7
8
9
10

Figure 9. Plots of Equation 3 $c = \frac{r(K_{a1} + K_1 r)}{(1 + \frac{K_1 r}{K_d})}$ with $K_d = 10$ and $K_1 = 1000$



120
110
100
90
80
70
60
50
40
30
20
10
0
0 1 2 3 4 5 6 7 8 9 10
Output variable c(t), volts
Input variable r(t), volts
$K_{a1} = 100$
$K_{a1} = 1$
$K_{a1} = 10$
Calculated values shown by solid lines.
Experimental points are
⊙ for $K_{a1} = 1$
▫ for $K_{a1} = 10$
△ for $K_{a1} = 100$

Donner Model 3500 analog computer.

Comparison of the three approaches

When viewing a system from its terminal characteristics, it is quite often impossible to distinguish between an open-loop, a closed-loop, or an adaptive control space system. It should be emphasized that although the terminal characteristics may be identical, the internal configurations may be radically different. Two examples will make this more clear.

Example 1. Referring to Figure 10 a the transfer function of the network $E_2(s)/E_1(s)$ may be written as

$$\frac{E_2(s)}{E_1(s)} = \frac{1/Cs}{R + 1/Cs} = \frac{1}{1 + RCs} . \qquad 7$$

Now referring to Figure 10 b and using standard block diagram algebra for feedback control systems the ratio of the output to the input may be written as

$$\frac{E_2(s)}{E_1(s)} = \frac{1/RCs}{1 + 1/RCs} = \frac{1}{1 + RCs} . \qquad 8$$

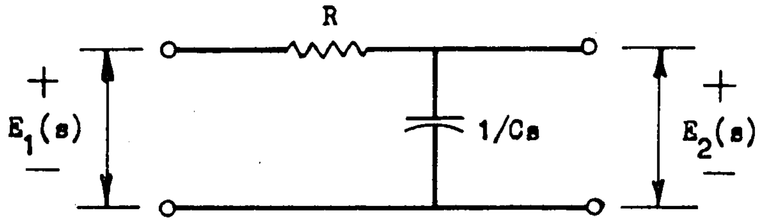
Notice that it is impossible to distinguish between the final expressions for the transfer function as given by Equations 7 and 8.

Example 2. An even more simple example may even make this more clear. Referring to Figure 11 a, the ratio of the output voltage v_2 to the input voltage v_1 may be written as

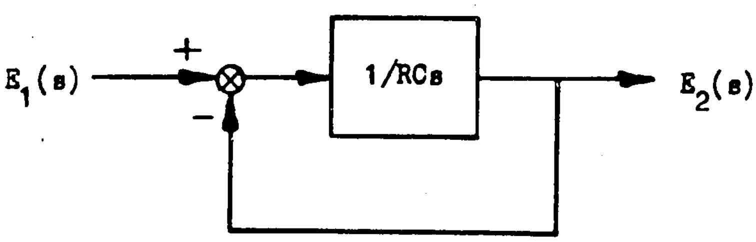
$$\frac{v_2}{v_1} = \frac{R_2}{R_1 + R_2} . \qquad 9$$

Now referring to Figure 11 b, and again using standard block diagram algebra, the ratio of the output to the input may be written as

Figure 10. The open-loop system shown in part a and the closed-loop system shown in part b both lead to the same transfer function $\frac{E_2(s)}{E_1(s)} = \frac{1}{1 + RCs}$

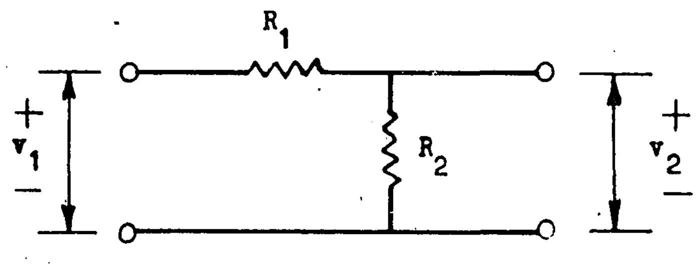


Part a

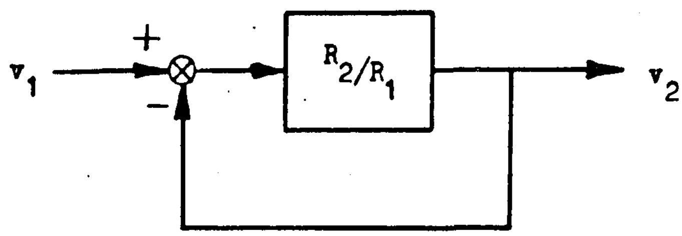


Part b

Figure 11. The open-loop system shown in part a and the closed-loop system shown in part b both lead to the same transfer characteristic $\frac{v_2}{v_1} = \frac{R_2}{R_1 + R_2}$



Part a



Part b

$$\frac{v_2}{v_1} = \frac{R_2/R_1}{1 + R_2/R_1} = \frac{R_2}{R_1 + R_2} . \qquad 10$$

Here again it is impossible to distinguish between a closed-loop and an open-loop system by just the terminal characteristics. The fundamental difference is the type of configuration inside.

A comparison of the results obtained by the open-loop, closed-loop, and adaptive control system approaches will now be made for the case where $K_o = K_1 = K_{a1} = 10$, and the desired gain $K_d = 10$. From Figure 3 for an open-loop system $c = 10r$; from Figure 5 for the closed-loop system $c = 10r$; and finally from Equation 1 or Figure 7, for the adaptive control system, again $c = 10r$. Notice that the terminal characteristic equation does not indicate the type of internal configuration that exists.

Computer Simulations

The simulation of any set of equations on an analog computer requires, in addition to the basic knowledge concerning the theory of analog computers, a detailed knowledge of the specific characteristics of the particular computer being used. Drift characteristics of the amplifiers determine whether time scaling is required; saturation levels of the amplifiers determine whether amplitude scaling is necessary; and such characteristics as amplifier phase shift and noise prevent the use of all theoretically possible operational amplifier configurations. Although many analog computer configurations are possible, two have been chosen because they are representative of what may be achieved.

Analog computer simulation 1

The functional block diagram for the analog computer solution of Equation 1 rewritten as

$$c = K_{a1}r + K_1r^2 - K_1rc/K_d \qquad 11$$

is shown in Figure 12. It is called a functional block diagram because it is simply an intermediate aid in the simulation and does not show detail such as the sign reversal introduced by every operational amplifier nor does it concern itself with maintaining proper signal amplitudes. Signal amplitudes which are too large exceed the maximum capabilities of the amplifiers while levels which are too small are noisy and result in low signal to noise ratios.

The wiring diagram for solving Equation 11 using approach 1 for $K_{a1} = K_1 = K_d = 10$ is shown in Figure 13 and the data obtained are plotted on Figure 7. Corresponding data are also plotted on Figures 8 and 9. This particular configuration is satisfactory for obtaining data of a static nature, but it has some shortcomings when used for determining the dynamic behavior of the system because of the manner in which adaption takes place. Two function multipliers are also required in approach 1 as compared to only one function multiplier in approach 2.

Analog computer simulation 2

The functional block diagram for the analog computer solution of

$$c = K_{a2}r + K_{a2}K_2r^2 - K_{a2}K_2rc/K_d \qquad 12$$

is shown in Figure 14. Equations 11 and 12 have exactly the same form and are of the same degree and order but different constants have been used.

The wiring diagram for solving Equation 12 is shown in Figure 15 for

Figure 12. Functional block diagram for an analog computer solution of Equation 1 rewritten as $c = K_{a1}r + K_1 r^2 - K_1 rc/K_d$

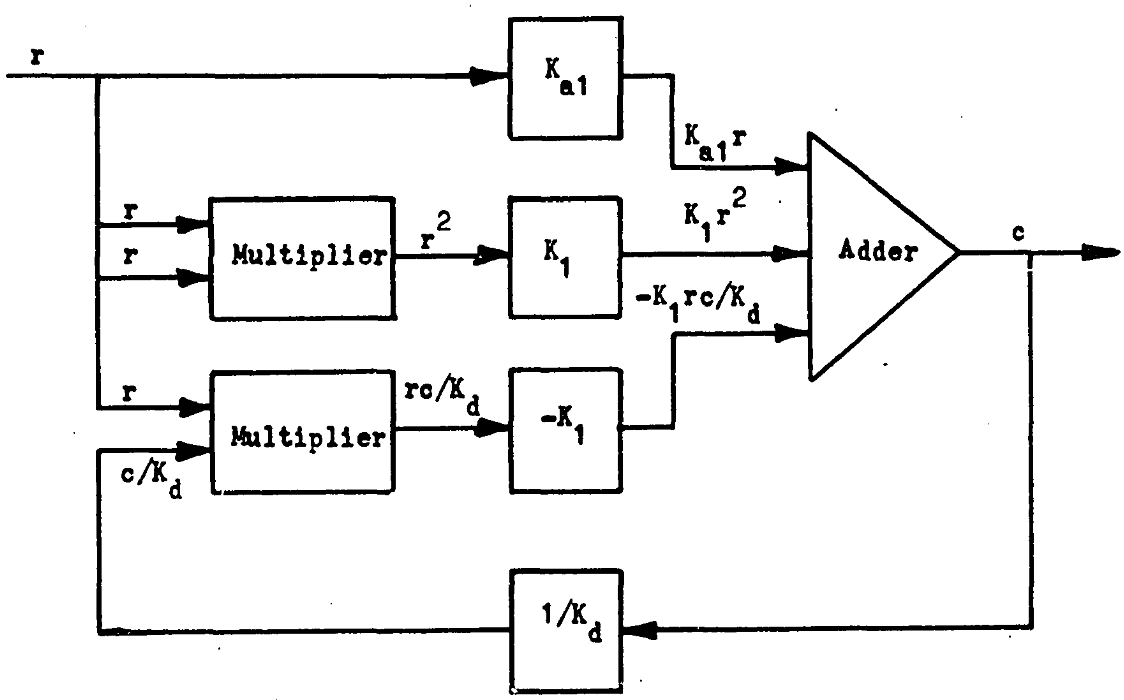
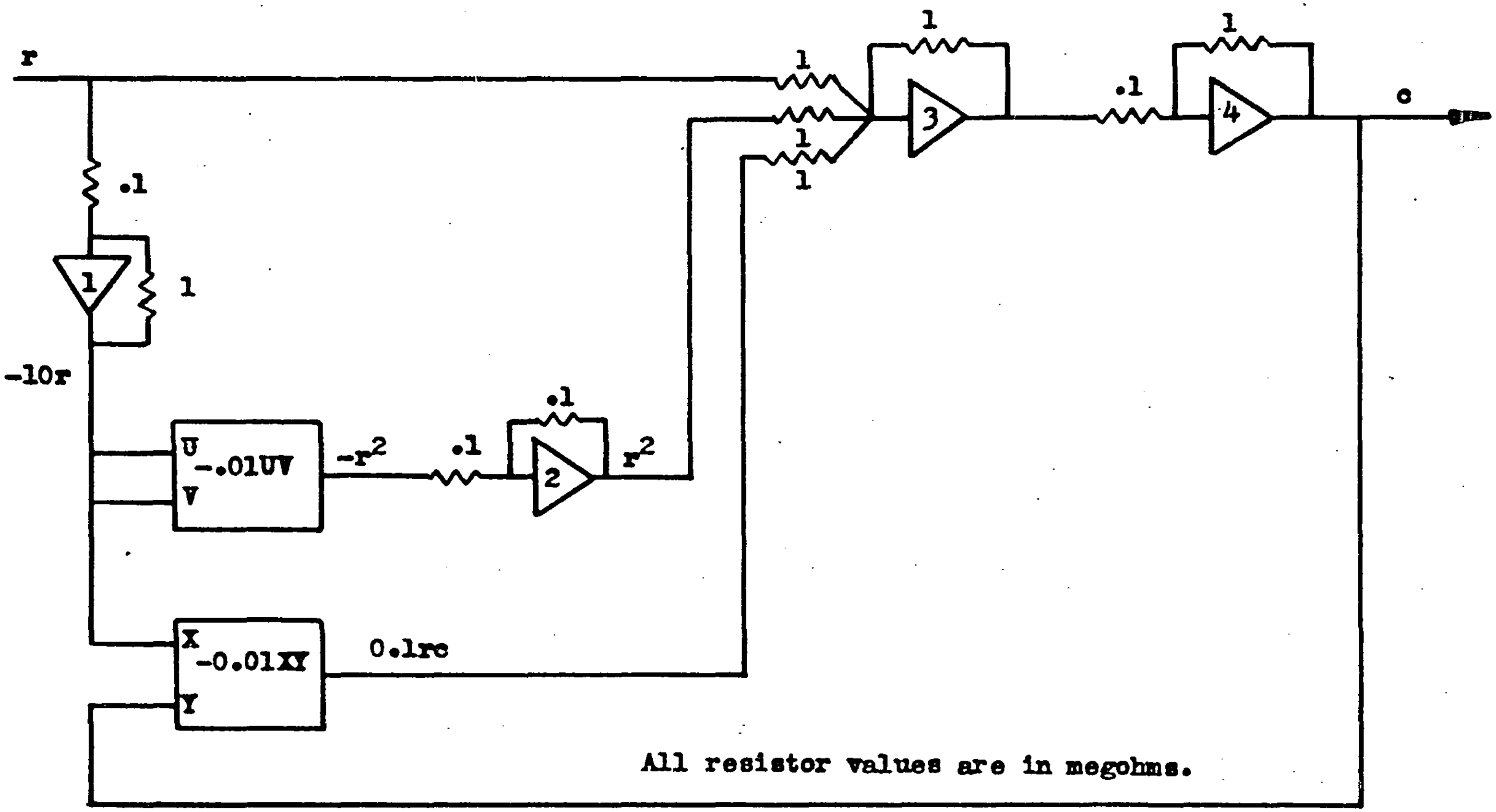
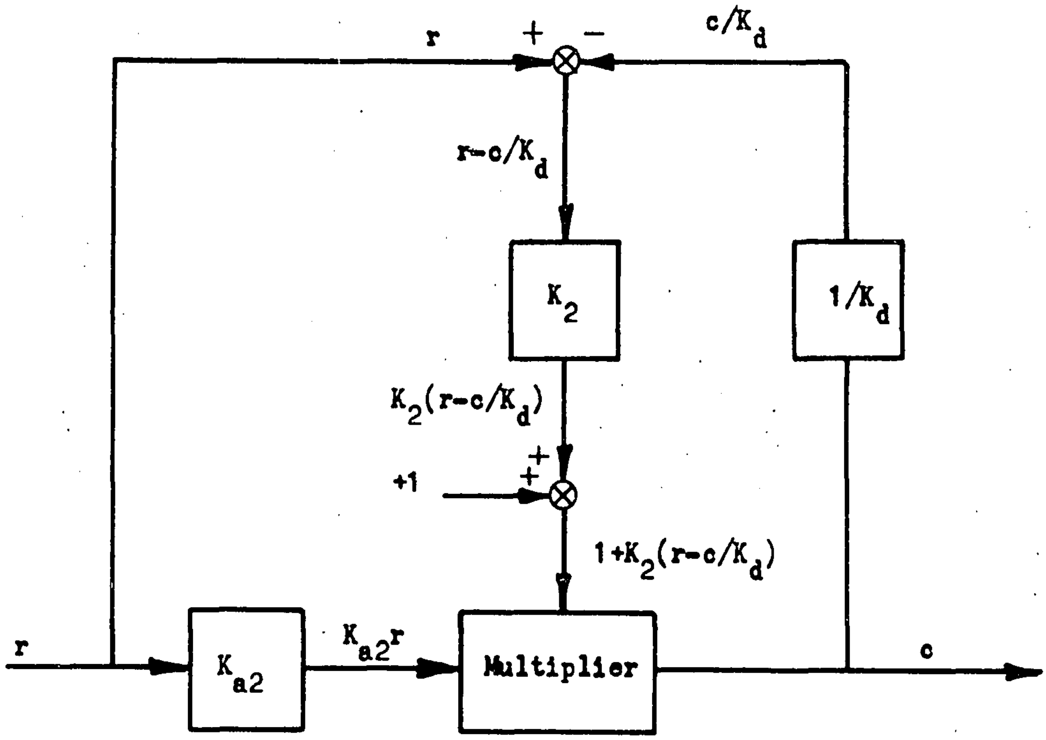

r
K_{a1}
$K_{a1}r$
r
r
Multiplier
r^2
K_1
K_1r^2
Adder
c
$-K_1rc/K_d$
r
Multiplier
rc/K_d
$-K_1$
c/K_d
$1/K_d$

Figure 13. Approach 1 to the Donner analog computer wiring diagram for solving the equation $0.1c = r + r^2 - 0.1rc$



All resistor values are in megohms.

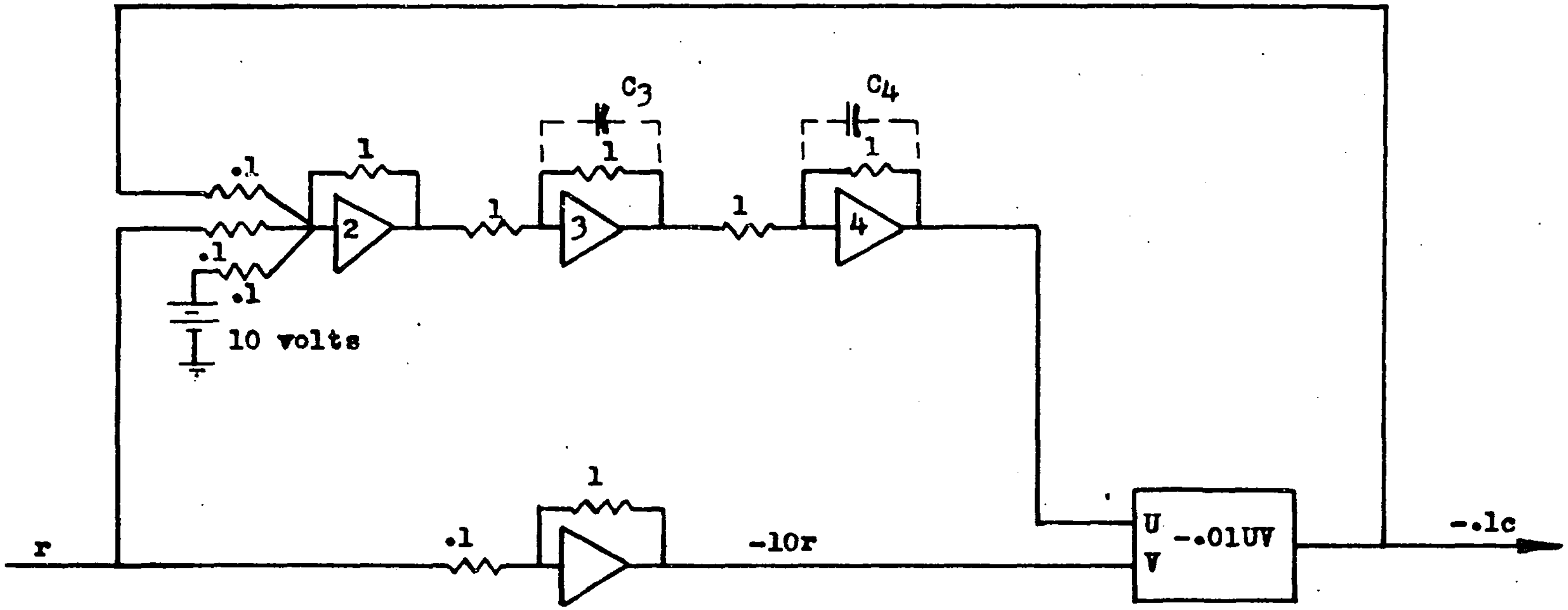
Figure 14. Functional block diagram of a typical reciprocal model reference adaptive control system represented by the equation $c = K_{a2}r + K_{a2}K_2r^2 - K_{a2}K_2rc/K_d$



r
+
−
c/K_d
r−c/K_d
K_2
1/K_d
K_2(r−c/K_d)
+
+
+1
1+K_2(r−c/K_d)
r
K_a2
K_a2 r
Multiplier
c

Figure 15. Approach 2 to the Donner analog wiring diagram for solving the equation $0.1c = r + r^2 - 0.1rc$

All resistor values are in megohms



$K_{a2} = K_d = 10$ and $K_2 = 1$. The data obtained from this setup are not significantly different from that obtained in Figure 13, but Figure 15 uses only one function multiplier and two of the four operational amplifiers are essentially spare amplifiers and are available for use as integrators or first order delays. The two spare amplifiers are numbers 3 and 4 which are connected for unity gain, and since each inverts, the output of amplifier 4 is equal to the input to amplifier 3.

Other analog computer simulations

There are literally hundreds of different possible analog computer configurations which will verify Equation 1 or some variation of it. It is the variations which are of much more interest now and will be discussed further.

To simulate a delay in adaption, a capacitor may be added in parallel to the feedback resistor associated with amplifier 3 in Figure 15. Similarly, if a capacitor is also added to amplifier 4, two cascaded first order delays may be simulated. Both of these cases have been simulated and agree with predicted results. To be more specific, in the case of one first order delay, the value of the output exponentially approaches the same final value as in the corresponding static case with a time constant equal to the product of the feedback capacitor and the feedback resistor.

To simulate an amplifier with limited bandwidth, one or two operational amplifiers may be inserted in Figure 13 in series with the output before it is fed back to amplifier 2. This situation has also been simulated in the laboratory but the adaptive characteristics of the system are represented by the measured steady state values which are identical to the data already presented and, therefore, they are not repeated here.

CONCLUSIONS AND RECOMMENDATIONS

The results of this study appear to substantiate the truth of the statement frequently made that there is not a unique solution to an engineering synthesis problem. These nonunique solutions are obtained by applying known approaches or viewpoints (such as feedback) toward their solution. Although the open-loop, closed-loop feedback, and adaptive control system approaches have been applied to a given problem, it would be presumptuous to assume that this exhausts all possibilities since some approaches are undoubtedly still to be discovered.

One salient conclusion of this dissertation is that although different approaches to a synthesis problem may lead to identical terminal characteristics, the internal configuration may be radically different. The optimum internal configuration is one then that utilizes the desirable characteristics to the utmost and minimizes the undesirable ones. For example, in conventional feedback, a higher than necessary (but varying gain) is exchanged for a lower but more constant one. In an adaptive control system, the stability problem is reduced by accepting the desired performance at a time later than would be provided by conventional feedback, for example.

It is not recommended that all systems henceforth be adaptive control systems because, they too have undesirable characteristics. When the complexity of the system is increased by adding first and second order time delays, system stability again becomes a problem and it is even more complicated than in the corresponding linear system because of its nonlinear nature. For example, the presence of a small damping ratio in a second order system and sinusoidal excitation will quite likely lead to the jump resonance phenomenon. Subharmonic generation is also possible. Although no general rules are presently available for defining the necessary conditions for its occurrence, it has been observed in lightly damped systems with nonlinear restoring forces. In all of the above cases, instability would most likely be observed by the presence of limit cycles (bounded oscillations) of both the stable and unstable type. Any one of the above situations could be (and has been) the subject of lengthy investigations in itself.

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